

Lorentz and CPT violation in QED revisited: A missing analysis

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Abstract

We investigate the breakdown of Lorentz symmetry in QED by a CPT violating interaction term consisting of the coupling of an axial fermion current with a constant vector field b , in the framework of algebraic renormalization – a regularization-independent method. We show, to all orders in perturbation theory, that a CPT-odd and Lorentz violating Chern-Simons-like term, definitively, is not radiatively induced by the axial coupling of the fermions with the constant vector b .

1 Introduction

The quantum electrodynamics (QED) with violation of Lorentz and CPT have been studied intensively in recent years. Among several issues, the possible generation of a Chern-Simons-like term induced by radiative corrections arising from a CPT and Lorentz violating term in the fermionic sector has been a recurrent theme in the literature. We particularly mention the following works [1]-[18] (and references cited therein), where many controversies have emerged from the discussion whether this Chern-Simons-like term could be generated by means of radiative

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corrections arising from the axial coupling of charged fermions to a constant vector b_μ responsible for the breakdown of Lorentz Symmetry.

In this paper, we reassess the discussion on the radiative generation of a Chern-Simons-like term induced from quantum corrections in the extended QED. Concerning to extended QED with a term which violates the Lorentz and CPT symmetries, most of the papers were devoted to discuss the gauge invariance of the model *only*, putting aside a more specific way how Lorentz invariance is broken. Here, we will discuss the latter point, giving attention to the requirement that the breakdown of Lorentz symmetry arising from the axial coupling of charged fermions to a constant vector b_μ be *soft* in the sense of Symanzik [19, 20, 21, 22], *i.e.*, has power-counting dimension less than four or, equivalently, is negligible in the deep Euclidean region of energy-momentum space. To the best of our knowledge this has not been investigated in details. In switching on the radiative corrections, it is a non-trivial task to study the effects of such a symmetry breaking. In particular, one has to ask how the corresponding Ward identity that characterizes the breaking behaves at the quantum level. Our aim is to show that, to the contrary of the claims found in the literature, radiative corrections arising from the axial coupling of charged fermions to a constant vector b_μ do not induce a Lorentz- and CPT-violating Chern-Simons-like term in the QED action.

2 Extended QED in the Classical Approximation

2.1 The Classical Theory

We start by considering an action for extended QED with a term which violates the Lorentz and CPT symmetries in the matter sector only. In the tree approximation, the classical action of extended QED with one Dirac spinor that we are considering here is given by:

$$\Sigma = \Sigma_S + \Sigma_{SB} + \Sigma_{IR} + \Sigma_{gf} , \quad (2.1)$$

where

$$\Sigma_S = \int d^4x \left\{ i\bar{\psi}\gamma^\mu(\partial_\mu + ieA_\mu)\psi - m\bar{\psi}\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} \right\} , \quad (2.2)$$

is the symmetric part of Σ under gauge and Lorentz transformations. The term

$$\Sigma_{SB} = - \int d^4x \, b_\mu \bar{\psi}\gamma_5\gamma^\mu\psi , \quad (2.3)$$

is the symmetry-breaking part of Σ that breaks the manifest Lorentz covariance by the presence of a constant vector b_μ which selects a preferential direction in Minkowski space-time, breaking its isotropy, as well as it breaks CPT.⁵

$$\Sigma_{IR} = \int d^4x \, \frac{1}{2}\lambda^2 A_\mu A^\mu , \quad (2.4)$$

⁵Greenberg proved that CPT invariance is necessary, but not sufficient, for Lorentz invariance [23].

is a mass term for the photon field,⁶ introduced in order to avoid infrared singularities and

$$\Sigma_{\text{gf}} = - \int d^4x \frac{1}{2\xi} (\partial_\mu A^\mu)^2 , \quad (2.5)$$

is a gauge-fixing action.

2.2 The Symmetries

2.2.1 Discrete Symmetries

The discrete symmetries of the theory are the following ones.

Charge Conjugation C: Assuming the Dirac representation of the γ -matrices [26], the charge conjugation transformations read:

$$\begin{aligned} \psi &\xrightarrow{C} \psi^c = C \bar{\psi}^t , \\ \bar{\psi} &\xrightarrow{C} \bar{\psi}^c = -\psi^t C^{-1} , \\ A_\mu &\xrightarrow{C} A_\mu^c = -A_\mu , \\ C\gamma_\mu C &= \gamma_\mu^t , \\ C\gamma_5 C &= -\gamma_5^t = -\gamma_5 . \end{aligned} \quad (2.6)$$

where C is the charge conjugation matrix, with $C^2 = -1$. All terms in the action Σ (2.1) are invariant under charge conjugation.

Parity P:

$$\begin{aligned} x &\xrightarrow{P} (x^0, -\vec{x}) , \\ \psi &\xrightarrow{P} \gamma^0 \psi , \\ \bar{\psi} &\xrightarrow{P} \bar{\psi} \gamma^0 , \\ A_\mu &\xrightarrow{P} A^\mu . \end{aligned} \quad (2.7)$$

All terms of the action are invariant, excepted the Lorentz breaking term Σ_{SB} (2.3), which transforms under parity as

$$\bar{\psi} b_\mu \gamma_5 \gamma^\mu \psi \xrightarrow{P} \begin{cases} -\bar{\psi} b_0 \gamma_5 \gamma^0 \psi \\ \bar{\psi} b_i \gamma_5 \gamma^i \psi , & (i = 1, 2, 3) . \end{cases} \quad (2.8)$$

⁶As we shall see, the gauge invariance properties are not spoiled by the photon mass: this is a peculiarity of the Abelian case [24]. This was studied in details for the QED in Ref.[25] using the BPHZ scheme.

Time Reversal T:

$$\begin{aligned}
\psi &\xrightarrow{T} T\psi , \\
\bar{\psi} &\xrightarrow{T} \bar{\psi}T , \\
A_\mu &\xrightarrow{T} A_\mu , \\
T\gamma^\mu T &= \gamma_\mu^T = \gamma^{\mu*} , \\
T\gamma_5 T &= \gamma_5 .
\end{aligned} \tag{2.9}$$

Under time reversal transformation, the broken Lorentz term, Σ_{SB} (2.3), transforms as below:

$$\bar{\psi}b_\mu\gamma_5\gamma^\mu\psi \xrightarrow{T} \begin{cases} \bar{\psi}b_0\gamma_5\gamma^0\psi \\ -\bar{\psi}b_i\gamma_5\gamma^i\psi, \quad (i = 1, 2, 3), \end{cases} \tag{2.10}$$

which implies time reversal violation, whereas the other terms in the action Σ (2.1) remain invariant.

Therefore, the action for extended QED, Σ (2.1), has CPT symmetry broken by the Lorentz breaking term, Σ_{SB} (2.3):

$$\bar{\psi}b_\mu\gamma_5\gamma^\mu\psi \xrightarrow{CPT} -\bar{\psi}b_\mu\gamma_5\gamma^\mu\psi . \tag{2.11}$$

2.2.2 Continuous Symmetries: The Functional Identities

The $U(1)$ gauge transformations are given by:

$$\begin{aligned}
\delta_g A_\mu(x) &= \frac{1}{e} \partial_\mu \omega(x) , \\
\delta_g \psi(x) &= -i \omega(x) \psi(x) , \\
\delta_g \bar{\psi}(x) &= i \omega(x) \bar{\psi}(x) ,
\end{aligned} \tag{2.12}$$

which are broken by the gauge-fixing and infrared regulator terms.

Subjected to the $U(1)$ gauge transformations (2.12), the action Σ (2.1) transforms as given by the following Ward identity:

$$\mathcal{W}_g \Sigma = -\frac{1}{e\xi} (\Box + e\xi\lambda^2) \partial_\mu A^\mu , \tag{2.13}$$

with the local Ward operator associated to the gauge transformations

$$\mathcal{W}_g = -\frac{1}{e} \partial_\mu \frac{\delta}{\delta A_\mu(x)} + i \bar{\psi}(x) \frac{\overrightarrow{\delta}}{\delta \bar{\psi}(x)} - i \frac{\overleftarrow{\delta}}{\delta \psi(x)} \psi(x) . \tag{2.14}$$

Note that the right-hand side of (2.13) being linear in the quantum field A_μ , will not be submitted to renormalization, *i.e.*, it will remain a classical breaking [20, 22].

On the other hand, the Lorentz symmetry is broken by the presence of the constant vector b_μ . The fields A_μ and ψ transform under infinitesimal Lorentz transformations $\delta x^\mu = \epsilon^\mu{}_\nu x^\nu$, with $\epsilon_{\mu\nu} = -\epsilon_{\nu\mu}$, as

$$\begin{aligned}\delta_L A_\mu &= -\epsilon^\lambda{}_\nu x^\nu \partial_\lambda A_\mu + \epsilon_\mu{}^\nu A_\nu \equiv \frac{1}{2} \epsilon^{\alpha\beta} \delta_{L\alpha\beta} A_\mu, \\ \delta_L \psi_\mu &= -\epsilon^\lambda{}_\nu x^\nu \partial_\lambda \psi - \frac{i}{4} \epsilon^{\mu\nu} \sigma_{\mu\nu} \psi \equiv \frac{1}{2} \epsilon^{\alpha\beta} \delta_{L\alpha\beta} \psi,\end{aligned}\tag{2.15}$$

where $\sigma_{\mu\nu} = \frac{i}{2} [\gamma_\mu, \gamma_\nu]$.

It should be noticed that the Lorentz breaking (2.3) is not linear in the dynamical fields, therefore will be renormalized. It is however a “*soft breaking*,” since its UV power-counting dimension⁷ is less than 4, namely 3. According to Symanzik [19, 20], a theory with soft symmetry breaking is renormalizable if the radiative corrections do not induce a breakdown of the symmetry by terms of UV power-counting dimension equal to 4 – called hard breaking terms. Concretely, according to the Weinberg theorem [27, 28, 24], this means that the symmetry of the theory in the asymptotic deep Euclidean region of momentum space is preserved by the radiative corrections. In order to control the Lorentz breaking and, in particular, its power-counting properties, following Symanzik [19, 20], and [29] for the specific case of Lorentz breaking, we introduce an external field $\beta_\mu(x)$, of dimension 1 and transforming under Lorentz transformations according to

$$\delta_L \beta_\mu(x) = -\epsilon^\lambda{}_\nu x^\nu \partial_\lambda \beta_\mu(x) + \epsilon_\mu{}^\nu (\beta_\nu(x) + b_\nu) \equiv \frac{1}{2} \epsilon^{\alpha\beta} \delta_{L\alpha\beta} \beta_\mu(x).\tag{2.16}$$

The functional operator which generates these transformations reads

$$\mathcal{W}_{L\alpha\beta} = \int d^4x \sum_{\varphi=A_\mu, \psi, \bar{\psi}, \beta} \delta_{L\alpha\beta} \varphi(x) \frac{\delta}{\delta \varphi(x)}.\tag{2.17}$$

Redefining the action by adding a term in β :

$$\tilde{\Sigma} = \Sigma - \int d^4x (\beta_\mu + b_\mu) \bar{\psi} \gamma^5 \gamma^\mu \psi,\tag{2.18}$$

one easily checks the classical Ward identity

$$\mathcal{W}_{L\alpha\beta} \tilde{\Sigma} = 0,\tag{2.19}$$

which, at $\beta_\mu = 0$, reduces to the broken Lorentz Ward identity

$$\mathcal{W}_{L\alpha\beta} \Sigma = \epsilon_\mu{}^\nu b_\nu \int d^4x \bar{\psi} \gamma^5 \gamma^\mu \psi.\tag{2.20}$$

The external field $\beta(x)$ being coupled to a gauge invariant expression, we take it to be gauge invariant in order to preserve gauge invariance,

$$\delta_g \beta_\mu(x) = 0.$$

Therefore, it follows that the action $\tilde{\Sigma}$ (2.18) satisfies the same gauge Ward identity (2.13) as the action Σ (2.1), namely:

$$\mathcal{W}_g \tilde{\Sigma} = -\frac{1}{e\xi} (\Box + e\xi\lambda^2) \partial_\mu A^\mu.\tag{2.21}$$

⁷The UV power-counting dimensions of A_μ and ψ are $d_A = 1$ and $d_\psi = \frac{3}{2}$.

3 Quantization

In this section, we present the perturbative quantization of the extended QED theory, using the algebraic renormalization procedure (see [22] for a review of the method and references to the original literature). Our aim is to prove that the full quantum theory has the same properties as the classical theory, *i.e.* prove that the Ward identities, associated to the gauge symmetry (2.21) and to the Lorentz symmetry (2.19), are satisfied to all orders of perturbation theory. In order to study the renormalizability of models characterized by a system of Ward identities, without referring to any special regularization procedure, two steps must be followed [22]: In the first step, we compute the possible anomalies of the Ward identities through an analysis of the Wess-Zumino consistency condition. Next, we check the stability of the classical action – which ensures that the quantum corrections do not produce counterterms corresponding to the renormalization of parameters not already present in the classical theory.

3.1 Wess-Zumino Condition: In Search for Anomalies

The perturbative expansion⁸ of the vertex functional:

$$\Gamma = \sum_{n \geq 0} \hbar \Gamma_n , \quad (3.1)$$

is such that it coincides with the classical action in the classical limit:

$$\Gamma = \Gamma_0 + \mathcal{O}(\hbar) , \quad (3.2)$$

where $\Gamma_0 = \tilde{\Sigma}$ (2.18).

We have to demonstrate that, at the quantum level, the theory fulfills perturbatively, to all orders, the gauge and Lorentz Ward identities:

$$\mathcal{W}_g \Gamma = -\frac{1}{e\xi} (\Box + e\xi\lambda^2) \partial_\mu A^\mu \quad (3.3)$$

and

$$\mathcal{W}_{L\alpha\beta} \Gamma = 0 , \quad (3.4)$$

– whose classical counterparts are given by (2.19) and (2.21) – together with the normalization conditions:

$$\begin{aligned} \Gamma_{\bar{\psi}\psi}(\not{p}) \Big|_{\not{p}=m} &= 0 , & \frac{\partial}{\partial \not{p}} \Gamma_{\bar{\psi}\psi}(\not{p}) \Big|_{\not{p}=m} &= 1 , \\ \frac{\partial}{\partial p^2} \Gamma_{A^T A^T}(p^2) \Big|_{p^2=\lambda^2} &= 1 , & -\frac{1}{4} \text{Tr}[\gamma^\mu \gamma^5 \Gamma_{\beta\mu} \bar{\psi}\psi(0, \not{p})] \Big|_{\not{p}=m} &= 1 . \end{aligned} \quad (3.5)$$

⁸Perturbation theory as usual is ordered according to the number of loops in the Feynman graphs or, equivalently, to the powers of \hbar .

These four normalization conditions define four of the seven parameters of the theory, namely the fermion mass m and the amplitudes of the fields ψ , A_μ and β_μ . The remaining parameters e , ξ , and λ^2 are defined as the coefficients appearing explicitly in the Ward identities (3.3) and (3.4).

We assume that an ultraviolet subtraction scheme, such as the BPHZ [28, 24], may be applied. It is well-known that the use of such a subtraction scheme may break the symmetries of the theory – this will certainly occur if no invariant regularization procedure is available. However, those possible breakings are fully governed by the Quantum Action Principle (QAP) [30, 24], which implies, here:

$$\mathcal{W}_g \Gamma + \frac{1}{e\xi} (\Box + e\xi\lambda^2) \partial_\mu A^\mu = \Delta_g \cdot \Gamma = \Delta_g + \mathcal{O}(\hbar) , \quad (3.6)$$

$$\mathcal{W}_{L\alpha\beta} \Gamma = \Delta_{L\alpha\beta} \cdot \Gamma = \Delta_{L\alpha\beta} + \mathcal{O}(\hbar) , \quad (3.7)$$

where Δ_g and $\Delta_{L\alpha\beta}$ are insertions with their UV dimensions bounded by $d_{\Delta_g} \leq 4$ and $d_{\Delta_{L\alpha\beta}} \leq 4$, respectively. The Ward identity operators \mathcal{W}_g and $\mathcal{W}_{L\alpha\beta}$ obey the following commutation rules:

$$[\mathcal{W}_g(x), \mathcal{W}_g(y)] = 0 , \quad (3.8)$$

$$[\mathcal{W}_g(x), \mathcal{W}_{L\alpha\beta}(y)] = 0 , \quad (3.9)$$

$$[\mathcal{W}_{L\alpha\beta}(x), \mathcal{W}_{L\gamma\delta}(y)] = \left\{ \eta_{\alpha\delta} \mathcal{W}_{L\beta\gamma}(x) + \eta_{\beta\gamma} \mathcal{W}_{L\alpha\delta}(x) - \eta_{\alpha\gamma} \mathcal{W}_{L\beta\delta}(x) - \eta_{\beta\delta} \mathcal{W}_{L\alpha\gamma}(x) \right\} \delta(x - y) , \quad (3.10)$$

where the latter commutation relation is that of the Lorentz algebra. By adopting the notation $[\alpha\beta] = i$ ($i = 1, \dots, 6$) for any (antisymmetric) pair of Lorentz indices of the Ward operator $\mathcal{W}_{L\alpha\beta}$, the eq.(3.10) can be rewritten as

$$[\mathcal{W}_{Li}(x), \mathcal{W}_{Lj}(y)] = f_{ij}^k \mathcal{W}_{Lk} \delta(x - y) . \quad (3.11)$$

Now, through (3.8), (3.9) and (3.10), the insertions Δ_g and Δ_{Li} appearing in (3.6) and (3.7), which are local field polynomials, satisfy the following Wess-Zumino consistency conditions:

$$\begin{aligned} \mathcal{W}_g(x) \Delta_g(y) - \mathcal{W}_g(y) \Delta_g(x) &= 0 , \\ \mathcal{W}_g(x) \Delta_{Lj}(y) - \mathcal{W}_{Lj}(y) \Delta_g(x) &= 0 , \\ \mathcal{W}_{Li}(x) \Delta_{Lj}(y) - \mathcal{W}_{Lj}(y) \Delta_{Li}(x) &= f_{ij}^k \Delta_{Lk}(x) \delta(x - y) . \end{aligned} \quad (3.12)$$

In the present case, it turns out to be convenient to proceed step by step, beginning with the Lorentz Ward identity. Its validity has been proved in [29], using Whitehead's lemma for semi-simple Lie groups, shown *e.g.* in [21], which states the vanishing of the first cohomology of such groups. In our context, this means that the general solution of the last of equations (3.12) has the form

$$\Delta_{Li}(x) = \mathcal{W}_{Li}(x) \hat{\Delta}_L , \quad (3.13)$$

with $\hat{\Delta}_L$ an integrated local insertion of UV dimension bounded by $d_{\hat{\Delta}_L} \leq 4$. $\hat{\Delta}_L$ can then be reabsorbed in the action as a – noninvariant – counterterm, order by order, thus establishing the Lorentz Ward identity (3.4) perturbatively at each order.

Let us now turn to the gauge Ward identity (3.3). Since we can now assume the validity of the Lorentz Ward identity, the consistency equations (3.12) reduce to

$$\begin{aligned}\mathcal{W}_g(x)\Delta_g(y) - \mathcal{W}_g(y)\Delta_g(x) &= 0, \\ \mathcal{W}_{Lj}(x)\Delta_g(y) &= 0.\end{aligned}\tag{3.14}$$

The general solution is well-known [31, 22]: it is the (Abelian) Adler-Bardeen-Bell-Jackiw anomaly [32] – up to terms which are gauge variations of integrated local insertions $\hat{\Delta}$ which can be reabsorbed as counterterms:

$$\Delta_g(x) = \mathcal{W}_g(x)\hat{\Delta} + r \epsilon_{\mu\nu\rho\lambda} F^{\mu\nu} F^{\rho\lambda}(x).\tag{3.15}$$

The anomaly coefficient r being not renormalized [33, 22], *i.e.*, it is zero if it vanishes at the 1 loop order, it suffices to check its vanishing at this order. But this is obvious, since the potentially dangerous axial current $j_5^\mu = \bar{\psi}\gamma_5\gamma^\mu\psi$ is only coupled to the external field β_μ – and not to any quantum field of the theory, which means that in fact no gauge anomaly can be produced [24, 10, 11]. Thus, the gauge Ward identity is preserved at the quantum level.

3.2 Stability: In Search for Counterterms

For the quantum theory the stability corresponds to the fact that the radiative corrections – the Ward identities being supposed to hold at this stage – can be reabsorbed by a redefinition of the initial parameters of the theory. As it is well known [22], it suffices to check the stability of the invariant classical action. In order to do so, one perturbs the action $\tilde{\Sigma}$ (2.18) by an arbitrary integrated local $\tilde{\Sigma}^c$:

$$\hat{\Sigma} = \tilde{\Sigma} + \epsilon \tilde{\Sigma}^c,\tag{3.16}$$

where ϵ is an infinitesimal parameter and the functional $\tilde{\Sigma}^c$ has the same quantum numbers (dimension, discrete symmetries) as the classical action. One then requires the deformed action $\hat{\Sigma}$ to obey all the classical Ward identities:

$$\mathcal{W}_g(\tilde{\Sigma} + \epsilon \tilde{\Sigma}^c) = \mathcal{W}_g(\tilde{\Sigma}) + \epsilon \mathcal{W}_g \tilde{\Sigma}^c = -\frac{1}{e\xi} (\Box + e\xi\lambda^2) \partial_\mu A^\mu,\tag{3.17}$$

and

$$\mathcal{W}_{L\alpha\beta}(\tilde{\Sigma} + \epsilon \tilde{\Sigma}^c) = \mathcal{W}_{L\alpha\beta}(\tilde{\Sigma}) + \epsilon \mathcal{W}_{L\alpha\beta} \tilde{\Sigma}^c = 0.\tag{3.18}$$

Then $\tilde{\Sigma}^c$ is subjected to the following set of constraints:

$$\mathcal{W}_g \tilde{\Sigma}^c = 0, \quad \mathcal{W}_{L\alpha\beta} \tilde{\Sigma}^c = 0,\tag{3.19}$$

therefore, the counterterm $\tilde{\Sigma}^c$ must be *symmetric* under the gauge and Lorentz symmetries as shown by eqs.(3.19), as well as invariant under charge conjugation.

The most general Lorentz invariant counterterm $\tilde{\Sigma}^c$, *i.e.*, the most general Lorentz invariant field polynomial with UV dimension bounded by $d \leq 4$ is given by an arbitrary superposition of the following – integrated – monomials:

$$\left\{ \bar{\psi}\psi, \bar{\psi}\gamma^\mu\partial_\mu\psi, \bar{\psi}\gamma^\mu A_\mu\psi, (\beta_\mu(x) + b_\mu)\bar{\psi}\gamma_5\gamma^\mu\psi, \partial_\mu A_\nu\partial^\mu A^\nu, \partial_\mu A_\nu\partial^\nu A^\mu, \right. \\ A_\mu A^\mu, A_\mu A^\mu A_\nu A^\nu, (\beta_\mu(x) + b_\mu)A^\mu\partial_\nu A^\nu, (\beta_\mu(x) + b_\mu)A^\nu\partial_\nu A^\mu, \\ \left. (\beta_\mu(x) + b_\mu)A_\nu\partial^\mu A^\nu, ((\beta_\mu(x) + b_\mu)A^\mu)^2, \epsilon_{\mu\nu\alpha\beta}(\beta^\mu(x) + b^\mu)A^\nu\partial^\alpha A^\beta \right\}.$$

Moreover, *gauge invariance* – represented by the first of Ward identities (3.19) – and the invariance under *charge conjugation* select the following four field polynomials:

$$\mathcal{P}_1 = i\bar{\psi}\gamma^\mu(\partial_\mu + ieA_\mu)\psi, \mathcal{P}_2 = \bar{\psi}\psi, \mathcal{P}_3 = (\beta_\mu(x) + b_\mu)\bar{\psi}\gamma_5\gamma^\mu\psi, \mathcal{P}_4 = F^{\mu\nu}F_{\mu\nu},$$

Now, it should be pointed out that, taking into account *only* the gauge Ward identity, $\mathcal{W}_g\tilde{\Sigma}^c = 0$, and the charge conjugation invariance, a Chern-Simons-like term, $\int d^4x \epsilon_{\mu\nu\alpha\beta}b^\mu A^\nu\partial^\alpha A^\beta$, could appear as a possible counterterm to the extended QED (2.18). It is softly broken Lorentz invariance as expressed by the second of Ward identities (3.19), which rules out this term.

Finally, the most general integrated local functional, $\tilde{\Sigma}^c$, satisfying the conditions of gauge and Lorentz invariances (3.19), and invariant under charge conjugation, is given by:

$$\tilde{\Sigma}^c = \int d^4x \sum_{i=1}^4 a_i \mathcal{P}_i(x), \quad (3.20)$$

where a_1, \dots, a_4 are arbitrary coefficients, fixed by the four normalization conditions (3.5) order by order in perturbation theory.

4 Conclusions

As a final conclusion, we proved through the use of the algebraic method of renormalization, which is independent of any kind of regularization scheme, that in the case of extended QED (2.1), to the contrary of some claims found out in the literature, a CPT-odd and Lorentz violating Chern-Simons-like term is definitively *not* generated by the radiative corrections. Therefore, if the Chern-Simons-like term is absent, from the beginning, at the classical level, it will be absent at the quantum level. This result has been obtained through a careful analysis of the consequences of the symmetries – Lorentz and gauge invariance – taken together in a consistent way.

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Lorentz and CPT violation in QED revisited: A missing analysis

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Abstract

We investigate the breakdown of Lorentz symmetry in QED by a CPT violating interaction term consisting of the coupling of an axial fermion current with a constant vector field b , in the framework of algebraic renormalization – a regularization-independent method. We show, to all orders in perturbation theory, that a CPT-odd and Lorentz violating Chern-Simons-like term, definitively, is not radiatively induced by the axial coupling of the fermions with the constant vector b .

1 Introduction

The quantum electrodynamics (QED) with violation of Lorentz and CPT have been studied intensively in recent years. Among several issues, the possible generation of a Chern-Simons-like term induced by radiative corrections arising from a CPT and Lorentz violating term in the fermionic sector has been a recurrent theme in the literature. We particularly mention the following works [1]-[18] (and references cited therein), where many controversies have emerged from the discussion whether this Chern-Simons-like term could be generated by means of radiative

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corrections arising from the axial coupling of charged fermions to a constant vector b_μ responsible for the breakdown of Lorentz Symmetry.

In this paper, we reassess the discussion on the radiative generation of a Chern-Simons-like term induced from quantum corrections in the extended QED. Concerning to extended QED with a term which violates the Lorentz and CPT symmetries, most of the papers were devoted to discuss the gauge invariance of the model *only*, putting aside a more specific way how Lorentz invariance is broken. Here, we will discuss the latter point, giving attention to the requirement that the breakdown of Lorentz symmetry arising from the axial coupling of charged fermions to a constant vector b_μ be *soft* in the sense of Symanzik [19, 20, 21, 22], *i.e.*, has power-counting dimension less than four or, equivalently, is negligible in the deep Euclidean region of energy-momentum space. To the best of our knowledge this has not been investigated in details. In switching on the radiative corrections, it is a non-trivial task to study the effects of such a symmetry breaking. In particular, one has to ask how the corresponding Ward identity that characterizes the breaking behaves at the quantum level. Our aim is to show that, to the contrary of the claims found in the literature, radiative corrections arising from the axial coupling of charged fermions to a constant vector b_μ do not induce a Lorentz- and CPT-violating Chern-Simons-like term in the QED action.

2 Extended QED in the Classical Approximation

2.1 The Classical Theory

We start by considering an action for extended QED with a term which violates the Lorentz and CPT symmetries in the matter sector only. In the tree approximation, the classical action of extended QED with one Dirac spinor that we are considering here is given by:

$$\Sigma = \Sigma_S + \Sigma_{SB} + \Sigma_{IR} + \Sigma_{gf} , \quad (2.1)$$

where

$$\Sigma_S = \int d^4x \left\{ i\bar{\psi}\gamma^\mu(\partial_\mu + ieA_\mu)\psi - m\bar{\psi}\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} \right\} , \quad (2.2)$$

is the symmetric part of Σ under gauge and Lorentz transformations. The term

$$\Sigma_{SB} = - \int d^4x \, b_\mu \bar{\psi}\gamma_5\gamma^\mu\psi , \quad (2.3)$$

is the symmetry-breaking part of Σ that breaks the manifest Lorentz covariance by the presence of a constant vector b_μ which selects a preferential direction in Minkowski space-time, breaking its isotropy, as well as it breaks CPT.⁵

$$\Sigma_{IR} = \int d^4x \, \frac{1}{2}\lambda^2 A_\mu A^\mu , \quad (2.4)$$

⁵Greenberg proved that CPT invariance is necessary, but not sufficient, for Lorentz invariance [23].

is a mass term for the photon field,⁶ introduced in order to avoid infrared singularities and

$$\Sigma_{\text{gf}} = - \int d^4x \frac{1}{2\xi} (\partial_\mu A^\mu)^2 , \quad (2.5)$$

is a gauge-fixing action.

2.2 The Symmetries

2.2.1 Discrete Symmetries

The discrete symmetries of the theory are the following ones.

Charge Conjugation C: Assuming the Dirac representation of the γ -matrices [26], the charge conjugation transformations read:

$$\begin{aligned} \psi &\xrightarrow{C} \psi^c = C \bar{\psi}^t , \\ \bar{\psi} &\xrightarrow{C} \bar{\psi}^c = -\psi^t C^{-1} , \\ A_\mu &\xrightarrow{C} A_\mu^c = -A_\mu , \\ C\gamma_\mu C &= \gamma_\mu^t , \\ C\gamma_5 C &= -\gamma_5^t = -\gamma_5 . \end{aligned} \quad (2.6)$$

where C is the charge conjugation matrix, with $C^2 = -1$. All terms in the action Σ (2.1) are invariant under charge conjugation.

Parity P:

$$\begin{aligned} x &\xrightarrow{P} (x^0, -\vec{x}) , \\ \psi &\xrightarrow{P} \gamma^0 \psi , \\ \bar{\psi} &\xrightarrow{P} \bar{\psi} \gamma^0 , \\ A_\mu &\xrightarrow{P} A^\mu . \end{aligned} \quad (2.7)$$

All terms of the action are invariant, excepted the Lorentz breaking term Σ_{SB} (2.3), which transforms under parity as

$$\bar{\psi} b_\mu \gamma_5 \gamma^\mu \psi \xrightarrow{P} \begin{cases} -\bar{\psi} b_0 \gamma_5 \gamma^0 \psi \\ \bar{\psi} b_i \gamma_5 \gamma^i \psi , & (i = 1, 2, 3) . \end{cases} \quad (2.8)$$

⁶As we shall see, the gauge invariance properties are not spoiled by the photon mass: this is a peculiarity of the Abelian case [24]. This was studied in details for the QED in Ref.[25] using the BPHZ scheme.

Time Reversal T:

$$\begin{aligned}
\psi &\xrightarrow{T} T\psi , \\
\bar{\psi} &\xrightarrow{T} \bar{\psi}T , \\
A_\mu &\xrightarrow{T} A_\mu , \\
T\gamma^\mu T &= \gamma_\mu^T = \gamma^{\mu*} , \\
T\gamma_5 T &= \gamma_5 .
\end{aligned} \tag{2.9}$$

Under time reversal transformation, the broken Lorentz term, Σ_{SB} (2.3), transforms as below:

$$\bar{\psi}b_\mu\gamma_5\gamma^\mu\psi \xrightarrow{T} \begin{cases} \bar{\psi}b_0\gamma_5\gamma^0\psi \\ -\bar{\psi}b_i\gamma_5\gamma^i\psi, \quad (i = 1, 2, 3), \end{cases} \tag{2.10}$$

which implies time reversal violation, whereas the other terms in the action Σ (2.1) remain invariant.

Therefore, the action for extended QED, Σ (2.1), has CPT symmetry broken by the Lorentz breaking term, Σ_{SB} (2.3):

$$\bar{\psi}b_\mu\gamma_5\gamma^\mu\psi \xrightarrow{CPT} -\bar{\psi}b_\mu\gamma_5\gamma^\mu\psi . \tag{2.11}$$

2.2.2 Continuous Symmetries: The Functional Identities

The $U(1)$ gauge transformations are given by:

$$\begin{aligned}
\delta_g A_\mu(x) &= \frac{1}{e} \partial_\mu \omega(x) , \\
\delta_g \psi(x) &= -i \omega(x) \psi(x) , \\
\delta_g \bar{\psi}(x) &= i \omega(x) \bar{\psi}(x) ,
\end{aligned} \tag{2.12}$$

which are broken by the gauge-fixing and infrared regulator terms.

Subjected to the $U(1)$ gauge transformations (2.12), the action Σ (2.1) transforms as given by the following Ward identity:

$$\mathcal{W}_g \Sigma = -\frac{1}{e\xi} (\Box + e\xi\lambda^2) \partial_\mu A^\mu , \tag{2.13}$$

with the Ward operator associated to the gauge transformations

$$\mathcal{W}_g(x) = -\frac{1}{e} \partial_\mu \frac{\delta}{\delta A_\mu(x)} + i \bar{\psi}(x) \frac{\overrightarrow{\delta}}{\delta \bar{\psi}(x)} - i \frac{\overleftarrow{\delta}}{\delta \psi(x)} \psi(x) . \tag{2.14}$$

Note that the right-hand side of (2.13) being linear in the quantum field A_μ , will not be submitted to renormalization, *i.e.*, it will remain a classical breaking [20, 22].

On the other hand, the Lorentz symmetry is broken by the presence of the constant vector b_μ . The fields A_μ and ψ transform under infinitesimal Lorentz transformations $\delta x^\mu = \epsilon^\mu{}_\nu x^\nu$, with $\epsilon_{\mu\nu} = -\epsilon_{\nu\mu}$, as

$$\begin{aligned}\delta_L A_\mu &= -\epsilon^\lambda{}_\nu x^\nu \partial_\lambda A_\mu + \epsilon_\mu{}^\nu A_\nu \equiv \frac{1}{2} \epsilon^{\alpha\beta} \delta_{L\alpha\beta} A_\mu, \\ \delta_L \psi &= -\epsilon^\lambda{}_\nu x^\nu \partial_\lambda \psi - \frac{i}{4} \epsilon^{\mu\nu} \sigma_{\mu\nu} \psi \equiv \frac{1}{2} \epsilon^{\alpha\beta} \delta_{L\alpha\beta} \psi,\end{aligned}\tag{2.15}$$

where $\sigma_{\mu\nu} = \frac{i}{2} [\gamma_\mu, \gamma_\nu]$.

It should be noticed that the Lorentz breaking (2.3) is not linear in the dynamical fields, therefore will be renormalized. It is however a “*soft breaking*,” since its UV power-counting dimension⁷ is less than 4, namely 3. According to Symanzik [19, 20], a theory with soft symmetry breaking is renormalizable if the radiative corrections do not induce a breakdown of the symmetry by terms of UV power-counting dimension equal to 4 – called hard breaking terms. Concretely, according to the Weinberg theorem [27, 28, 24], this means that the symmetry of the theory in the asymptotic deep Euclidean region of momentum space is preserved by the radiative corrections. In order to control the Lorentz breaking and, in particular, its power-counting properties, following Symanzik [19, 20], and [29] for the specific case of Lorentz breaking, we introduce an external field $\beta_\mu(x)$, of dimension 1 and transforming under Lorentz transformations according to

$$\delta_L \beta_\mu(x) = -\epsilon^\lambda{}_\nu x^\nu \partial_\lambda \beta_\mu(x) + \epsilon_\mu{}^\nu (\beta_\nu(x) + b_\nu) \equiv \frac{1}{2} \epsilon^{\alpha\beta} \delta_{L\alpha\beta} \beta_\mu(x).\tag{2.16}$$

The functional operator which generates these transformations reads

$$\mathcal{W}_{L\alpha\beta} = \int d^4x \sum_{\varphi=A_\mu, \psi, \bar{\psi}, \beta} \delta_{L\alpha\beta} \varphi(x) \frac{\delta}{\delta \varphi(x)}.\tag{2.17}$$

Redefining the action by adding a term in β_μ :

$$\tilde{\Sigma} = \Sigma - \int d^4x \beta_\mu \bar{\psi} \gamma_5 \gamma^\mu \psi,\tag{2.18}$$

one easily checks the classical Ward identity

$$\mathcal{W}_{L\alpha\beta} \tilde{\Sigma} = 0,\tag{2.19}$$

which, at $\beta_\mu = 0$, reduces to the broken Lorentz Ward identity

$$\mathcal{W}_{L\alpha\beta} \Sigma = \epsilon_\mu{}^\nu b_\nu \int d^4x \bar{\psi} \gamma_5 \gamma^\mu \psi.\tag{2.20}$$

The external field $\beta_\mu(x)$ being coupled to a gauge invariant expression (the axial current: $j_5^\mu = \bar{\psi} \gamma_5 \gamma^\mu \psi$), we take it to be gauge invariant in order to preserve gauge invariance,

$$\mathcal{W}_g \int d^4x \beta_\mu \bar{\psi} \gamma_5 \gamma^\mu \psi = 0 \implies \delta_g \beta_\mu(x) = 0.\tag{2.21}$$

Therefore, it follows that the action $\tilde{\Sigma}$ (2.18) satisfies the same gauge Ward identity (2.13) as the action Σ (2.1), namely:

$$\mathcal{W}_g \tilde{\Sigma} = -\frac{1}{e\xi} (\Box + e\xi\lambda^2) \partial_\mu A^\mu.\tag{2.22}$$

⁷The UV power-counting dimensions of A_μ and ψ are $d_A = 1$ and $d_\psi = \frac{3}{2}$.

3 Quantization

In this section, we present the perturbative quantization of the extended QED theory, using the algebraic renormalization procedure (see [22] for a review of the method and references to the original literature). Our aim is to prove that the full quantum theory has the same properties as the classical theory, *i.e.* prove that the Ward identities, associated to the gauge symmetry (2.22) and to the Lorentz symmetry (2.19), are satisfied to all orders of perturbation theory. In order to study the renormalizability of models characterized by a system of Ward identities, without referring to any special regularization procedure, two steps must be followed [22]: In the first step, we compute the possible anomalies of the Ward identities through an analysis of the Wess-Zumino consistency condition. Next, we check the stability of the classical action – which ensures that the quantum corrections do not produce counterterms corresponding to the renormalization of parameters not already present in the classical theory.

3.1 Wess-Zumino Condition: In Search for Anomalies

The perturbative expansion⁸ of the vertex functional:

$$\Gamma = \sum_{n \geq 0} \hbar \Gamma_n , \quad (3.1)$$

is such that it coincides with the classical action in the classical limit:

$$\Gamma = \Gamma_0 + \mathcal{O}(\hbar) , \quad (3.2)$$

where $\Gamma_0 = \tilde{\Sigma}$ (2.18).

We have to demonstrate that, at the quantum level, the theory fulfills perturbatively, to all orders, the gauge and Lorentz Ward identities:

$$\mathcal{W}_g \Gamma = -\frac{1}{e\xi} (\Box + e\xi\lambda^2) \partial_\mu A^\mu \quad (3.3)$$

and

$$\mathcal{W}_{L\alpha\beta} \Gamma = 0 , \quad (3.4)$$

– whose classical counterparts are given by (2.19) and (2.22) – together with the normalization conditions:

$$\begin{aligned} \Gamma_{\bar{\psi}\psi}(\not{p}) \Big|_{\not{p}=m} &= 0 , & \frac{\partial}{\partial \not{p}} \Gamma_{\bar{\psi}\psi}(\not{p}) \Big|_{\not{p}=m} &= 1 , \\ \frac{\partial}{\partial p^2} \Gamma_{A^T A^T}(p^2) \Big|_{p^2=\lambda^2} &= 1 , & -\frac{1}{4} \text{Tr}[\gamma^\mu \gamma^5 \Gamma_{\beta\mu} \bar{\psi}\psi(0, \not{p})] \Big|_{\not{p}=m} &= 1 . \end{aligned} \quad (3.5)$$

⁸Perturbation theory as usual is ordered according to the number of loops in the Feynman graphs or, equivalently, to the powers of \hbar .

These four normalization conditions define four of the seven parameters of the theory, namely the fermion mass m and the amplitudes of the fields ψ , A_μ and β_μ . The remaining parameters e , ξ , and λ^2 are defined as the coefficients appearing explicitly in the Ward identities (3.3) and (3.4).

We assume that an ultraviolet subtraction scheme, such as the BPHZ [28, 24], may be applied. It is well-known that the use of such a subtraction scheme may break the symmetries of the theory – this will certainly occur if no invariant regularization procedure is available. However, those possible breakings are fully governed by the Quantum Action Principle (QAP) [30, 24], which implies, here:

$$\mathcal{W}_g \Gamma + \frac{1}{e\xi} (\square + e\xi\lambda^2) \partial_\mu A^\mu = \Delta_g \cdot \Gamma = \Delta_g + \mathcal{O}(\hbar) , \quad (3.6)$$

$$\mathcal{W}_{L\alpha\beta} \Gamma = \Delta_{L\alpha\beta} \cdot \Gamma = \Delta_{L\alpha\beta} + \mathcal{O}(\hbar) , \quad (3.7)$$

where $\Delta_g(x)$ and $\Delta_{L\alpha\beta}(x)$ are local insertions with their UV dimensions bounded by $d_{\Delta_g} \leq 4$ and $d_{\Delta_{L\alpha\beta}} \leq 4$, respectively. The Ward identity operators \mathcal{W}_g and $\mathcal{W}_{L\alpha\beta}$ obey the following commutation rules:

$$[\mathcal{W}_g(x), \mathcal{W}_g(y)] = 0 , \quad (3.8)$$

$$[\mathcal{W}_g(x), \mathcal{W}_{L\alpha\beta}(y)] = 0 , \quad (3.9)$$

$$[\mathcal{W}_{L\alpha\beta}(x), \mathcal{W}_{L\gamma\delta}(y)] = \left\{ \eta_{\alpha\delta} \mathcal{W}_{L\beta\gamma}(x) + \eta_{\beta\gamma} \mathcal{W}_{L\alpha\delta}(x) - \eta_{\alpha\gamma} \mathcal{W}_{L\beta\delta}(x) - \eta_{\beta\delta} \mathcal{W}_{L\alpha\gamma}(x) \right\} \delta(x-y) , \quad (3.10)$$

where the latter commutation relation is that of the Lorentz algebra. By adopting the notation $[\alpha\beta] = i$ ($i = 1, \dots, 6$) for any (antisymmetric) pair of Lorentz indices of the Ward operator $\mathcal{W}_{L\alpha\beta}$, the eq.(3.10) can be rewritten as

$$[\mathcal{W}_{Li}(x), \mathcal{W}_{Lj}(y)] = f_{ij}^k \mathcal{W}_{Lk} \delta(x-y) . \quad (3.11)$$

Now, through (3.8), (3.9) and (3.10), the insertions Δ_g and Δ_{Li} appearing in (3.6) and (3.7), which are local field polynomials, satisfy the following Wess-Zumino consistency conditions:

$$\begin{aligned} \mathcal{W}_g(x) \Delta_g(y) - \mathcal{W}_g(y) \Delta_g(x) &= 0 , \\ \mathcal{W}_g(x) \Delta_{Lj}(y) - \mathcal{W}_{Lj}(y) \Delta_g(x) &= 0 , \\ \mathcal{W}_{Li}(x) \Delta_{Lj}(y) - \mathcal{W}_{Lj}(y) \Delta_{Li}(x) &= f_{ij}^k \Delta_{Lk}(x) \delta(x-y) . \end{aligned} \quad (3.12)$$

In the present case, it turns out to be convenient to proceed step by step, beginning with the Lorentz Ward identity. Its validity has been proved in [29], using Whitehead's lemma for semi-simple Lie groups, shown *e.g.* in [21], which states the vanishing of the first cohomology of such groups. In our context, this means that the general solution of the last of equations (3.12) has the form

$$\Delta_{Li}(x) = \mathcal{W}_{Li}(x) \hat{\Delta}_L , \quad (3.13)$$

with $\hat{\Delta}_L$ an integrated local insertion of UV dimension bounded by $d_{\hat{\Delta}_L} \leq 4$. $\hat{\Delta}_L$ can then be reabsorbed in the action as a – noninvariant – counterterm, order by order, thus establishing the Lorentz Ward identity (3.4) perturbatively at each order.

Let us now turn to the gauge Ward identity (3.3). Since we can now assume the validity of the Lorentz Ward identity, the consistency equations (3.12) reduce to

$$\begin{aligned}\mathcal{W}_g(x)\Delta_g(y) - \mathcal{W}_g(y)\Delta_g(x) &= 0, \\ \mathcal{W}_{Lj}(x)\Delta_g(y) &= 0.\end{aligned}\tag{3.14}$$

The general solution is well-known [31, 22]: it is the (Abelian) Adler-Bardeen-Bell-Jackiw anomaly [32] – up to terms which are gauge variations of integrated local insertions $\hat{\Delta}$ which can be reabsorbed as counterterms:

$$\Delta_g(x) = \mathcal{W}_g(x)\hat{\Delta} + r \epsilon_{\mu\nu\rho\lambda} F^{\mu\nu} F^{\rho\lambda}(x).\tag{3.15}$$

The anomaly coefficient r being not renormalized [33, 22], *i.e.*, it is zero if it vanishes at the 1 loop order, it suffices to check its vanishing at this order. But this is obvious, since the potentially dangerous axial current $j_5^\mu = \bar{\psi}\gamma_5\gamma^\mu\psi$ is only coupled to the external field β_μ – and not to any quantum field of the theory, which means that in fact no gauge anomaly can be produced [24, 10, 11]. Thus, the gauge Ward identity is preserved at the quantum level.

3.2 Stability: In Search for Counterterms

For the quantum theory the stability corresponds to the fact that the radiative corrections – the Ward identities being supposed to hold at this stage – can be reabsorbed by a redefinition of the initial parameters of the theory. As it is well known [22], it suffices to check the stability of the invariant classical action. In order to do so, one perturbs the action $\tilde{\Sigma}$ (2.18) by an arbitrary integrated local $\tilde{\Sigma}^c$:

$$\hat{\Sigma} = \tilde{\Sigma} + \epsilon \tilde{\Sigma}^c,\tag{3.16}$$

where ϵ is an infinitesimal parameter and the functional $\tilde{\Sigma}^c$ has the same quantum numbers (dimension, discrete symmetries) as the classical action. One then requires the deformed action $\hat{\Sigma}$ to obey all the classical Ward identities:

$$\mathcal{W}_g(\tilde{\Sigma} + \epsilon \tilde{\Sigma}^c) = \mathcal{W}_g(\tilde{\Sigma}) + \epsilon \mathcal{W}_g \tilde{\Sigma}^c = -\frac{1}{e\xi} (\Box + e\xi\lambda^2) \partial_\mu A^\mu,\tag{3.17}$$

and

$$\mathcal{W}_{L\alpha\beta}(\tilde{\Sigma} + \epsilon \tilde{\Sigma}^c) = \mathcal{W}_{L\alpha\beta}(\tilde{\Sigma}) + \epsilon \mathcal{W}_{L\alpha\beta} \tilde{\Sigma}^c = 0.\tag{3.18}$$

Then $\tilde{\Sigma}^c$ is subjected to the following set of constraints:

$$\mathcal{W}_g \tilde{\Sigma}^c = 0, \quad \mathcal{W}_{L\alpha\beta} \tilde{\Sigma}^c = 0,\tag{3.19}$$

therefore, the counterterm $\tilde{\Sigma}^c$ must be *symmetric* under the gauge and Lorentz symmetries as shown by eqs.(3.19), as well as invariant under charge conjugation.

The most general Lorentz invariant counterterm $\tilde{\Sigma}^c$, *i.e.*, the most general Lorentz invariant field polynomial with UV dimension bounded by $d \leq 4$ is given by an arbitrary superposition of the following – *integrated* – monomials:

$$\left\{ \bar{\psi}\psi , \bar{\psi}\gamma^\mu\partial_\mu\psi , \bar{\psi}\gamma^\mu A_\mu\psi , (\beta_\mu(x) + b_\mu)\bar{\psi}\gamma_5\gamma^\mu\psi , \partial_\mu A_\nu\partial^\mu A^\nu , \partial_\mu A_\nu\partial^\nu A^\mu , \right. \\ A_\mu A^\mu , A_\mu A^\mu A_\nu A^\nu , (\beta_\mu(x) + b_\mu)A^\mu\partial_\nu A^\nu , (\beta_\mu(x) + b_\mu)A^\nu\partial_\nu A^\mu , \\ \left. (\beta_\mu(x) + b_\mu)A_\nu\partial^\mu A^\nu , ((\beta_\mu(x) + b_\mu)A^\mu)^2 , \epsilon_{\mu\nu\alpha\beta}(\beta^\mu(x) + b^\mu)A^\nu\partial^\alpha A^\beta \right\} .$$

Moreover, *gauge invariance* – represented by the first of Ward identities (3.19) – and the invariance under *charge conjugation* select the following four field polynomials:

$$\mathcal{P}_1 = i\bar{\psi}\gamma^\mu(\partial_\mu + ieA_\mu)\psi , \mathcal{P}_2 = \bar{\psi}\psi , \mathcal{P}_3 = (\beta_\mu(x) + b_\mu)\bar{\psi}\gamma_5\gamma^\mu\psi , \mathcal{P}_4 = F^{\mu\nu}F_{\mu\nu} ,$$

Now, it should be pointed out that, taking into account *only* the gauge Ward identity, $\mathcal{W}_g\tilde{\Sigma}^c = 0$, and the charge conjugation invariance, a Chern-Simons-like term, $\int d^4x \epsilon_{\mu\nu\alpha\beta}b^\mu A^\nu\partial^\alpha A^\beta$, could appear as a possible counterterm to the extended QED (2.18). It is softly broken Lorentz invariance as expressed by the second of Ward identities (3.19), which rules out this term. More precisely, it is a consequence of the postulated gauge invariance of the external field $\beta_\mu(x)$ introduced to characterize the softly broken Lorentz invariance.

Finally, the most general integrated local functional, $\tilde{\Sigma}^c$, satisfying the conditions of gauge and Lorentz invariances (3.19), and invariant under charge conjugation, is given by:

$$\tilde{\Sigma}^c = \int d^4x \sum_{i=1}^4 a_i \mathcal{P}_i(x) , \quad (3.20)$$

where a_1, \dots, a_4 are arbitrary coefficients, fixed by the four normalization conditions (3.5) order by order in perturbation theory.

4 Conclusions

We proved through the use of the algebraic method of renormalization, which *is independent of any kind of regularization scheme*, that in the case of extended QED (2.1), under the hypothesis discussed in the next paragraph, a CPT-odd and Lorentz violating Chern-Simons-like term is definitively *not* generated by the radiative corrections. Therefore, if the Chern-Simons-like term is absent, from the beginning, at the classical level, it will be absent at the quantum level. This result has been obtained through a careful analysis of the consequences of the symmetries – Lorentz and gauge invariance – taken together in a consistent way.

As we have said in the introduction, various and apparently contradictory claims are found in the literature. We must stress that our result is linked to an assumption we have made, namely that the external vector field $\beta_\mu(x)$, introduced in order to control the Lorentz breaking, *is gauge invariant*. Equivalently, our hypothesis has been that the axial current to which $\beta_\mu(x)$ is coupled and which characterizes the Lorentz breaking, is gauge invariant. As discussed in details in [2] with the help of explicit one-loop computations, this choice naturally forbids a Chern-Simons like counterterm – whose integrand is not gauge invariant – if this term is not already present in the tree approximation. Our work confirms this point, unambiguously, to *all orders of perturbation theory*. Note that, still according to [2], relaxing the assumption of gauge invariance of the local axial current and only requiring the invariance of its spacetime integral, would allow such a counterterm – and even fix it in a so-called “nonperturbative in b_ν ” treatment as the authors of [2] show in the one-loop approximation.

As a final remark, it should be noted that the same vanishing result has been proven to all orders in Ref. [10], with similar methods but *no explicit use* of the Lorentz Ward identities. There it is argued that, if the theory is correctly defined through Ward identities and normalization conditions, no Chern-Simons-like term appears, without any ambiguity. This is related to the fact that such term, bilinear in the gauge field, appears in fact as a *minor modification* to the gauge-fixing term. Then, as part of the “gauge term,” it is not renormalized. However, in our opinion, this argument must be better understood, since the analysis contained in Ref. [34] (using the method of spin projectors) shows that the Chern-Simons-like term is linked to the sector of spin 1, which is the sector that carries the physical degrees of freedom of the model. This apparently indicates that the Chern-Simons-like term could not be seen as a minor modification to the term of gauge-fixing [35].

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